Wave Optics and Multipath in the Impact Parameter Domain

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Topics

- Motivation
- Impact multipath in GRAS data
- How it looks like in the phase transform
- A Multipath Index
- (Very) preliminary regional distribution
- Summary
Motivation

- A few years ago, we did a study on different wave optics retrieval methodologies (with DMI, GFZ, UCAR, RUAG, …) because we wanted to learn what would be the “best” algorithm – FSI, Phase Transform, CT, CT2, FIOs, …
- Being used to the highly consistent RO retrievals in the stratosphere, we were surprised about the spread of results in tropospheric retrievals; the amount of structural uncertainty between processors was huge.
- We finally rolled our own FSI version, but for a long time struggled with occultations - especially the deep ones.

- In 2010, Sergey published a paper on the role of noise in wave optics retrievals, showing that
  - not using good data deep down would cause negative biases, but
  - not cutting off noise might cause positive biases in the troposphere.
Atmospheric Multipath

Strong vertical gradients (in humidity) may cause strong bending on the top side...

More advanced retrievals (aka wave optics) and enhanced measurement modes required

...resulting in crossing rays and multiple tones (doppler) at the same time
Wave Optics

All wave optics methods, in one way or another...

- ...map the signal into the impact parameter domain...

  – because –

- ...under spherical symmetry, impact parameter is a constant along each ray...
  - ...and therefore rays can’t cross...
  - ...and therefore there is no multipath...
  - ...so problem solved!

Note: spherical symmetry = no horizontal gradients
- Reassigned time-frequency spectrum of GRAS signal mapped to impact parameter/bending angle
- Grey lines indicate SLTA – multiple signal components around -70 km SLTA as well as reflections on sea or ice
- Wave optics retrieval (black solid line) works very well
- Bending angle peak cut-off due to vertical smoothing
Where to cut off? Use amplitude (or SNR)!

- Note: SNR of features contributing to the multipath are well above GRAS noise level
- GRAS tracks C/A code even in open loop, so we are measuring *something*
- And my apologies – Open Loop SNRs plotted with a vertical offset... :-(

- There is no immediate reason where to cut “the noise” off, so we experimented...
When to cut off?

- 05:05:05
- 05:05:15
- 05:05:08
- 05:05:20
- 05:05:12
- full signal
Let’s look at the spectrum a bit closer...

- Ordinary spectrogram (top)
- Reassigned spectrogram (bottom)
- Multiple beams – maxima – at the same impact height - impact multipath!
- Commonly assumed to occur if spherical symmetry is violated – a fundamental assumption in RO.
- Current wave optics methods fail in this circumstances, just as GO fails for atmospheric multipath.
Geometrical vs. Wave Optics – Conceptionally

- Geometrical optics: considers signal in the time domain,

\[ S(t) = A(t) e^{j\phi(t)} \]

- calculates instantaneous frequency

\[ \omega(t) = \frac{d\phi(t)}{dt} \]

- and from that one impact parameters and bending angles using the Doppler equation and geometry of the occultation.

- Wave optics transforms the signal from the time domain into the impact parameter domain using the Doppler equation and geometry to obtain

\[ \hat{S}(p) = \hat{A}(p) e^{j\hat{\phi}(p)} \]

- and then calculates bending angle via something like

\[ \alpha(p) = -\frac{d\hat{\phi}(p)}{dp} \]
Recapping Multipath – Cohen & Lee (1988)

\[ s(t) = s_1(t) + s_2(t) = A_1(t) e^{j\varphi_1(t)} + A_2(t) e^{j\varphi_2(t)} = A(t) e^{j\varphi(t)} \]

**Fig. 1 A-B.** Examples illustrating the behavior of the instantaneous frequency for the signal 
\[ s(t) = A_1 e^{10jt} + A_2 e^{20jt} \]. In A and B the amplitudes are \( A_1 = .2, A_2 = 1 \) and \( A_1 = -1.2, A_2 = 1 \), respectively.

- Nominal instantaneous frequency for multi-component signals may by far exceed the range of the original frequencies, and is not related to any of them (well. You can’t extract them easily...)
- Consider \( t \to p \) and \( \omega \to -\alpha \) which covers wave optics...
Identifying Impact Multipath

- Spectra (blue) and reassigned spectra (red) sampled at constant impact heights
- Error bars: 1\textsuperscript{st} (mean) and 2\textsuperscript{nd} (sdev) moment
- Note that spectral lines are still relatively well confined, while instantaneous frequency may vary much more.
Spectral Bandwidth for Error Estimates?

- Pioneered for RO by Hocke et al. (1999), and later by Gorbunov et al. (2006):
  - Use the width of spectral beam as proxy for random error of the bending angle/impact parameter

- Cohen et al. (late ‘80s, early 90’s):
  - Ridges of time-frequency distribution do indeed provide estimates for instantaneous frequency of a signal (true for all TFAs)
  - Also introduced the concept of an instantaneous bandwidth (i.e., the width of a beam)
  - The arguments can be translated analogously to group delay / FSI / wave optics retrievals

- However: instantaneous bandwidth of a TFA depends on the signal and on the windowing applied; see ordinary vs. reassigned spectra in the previous examples.
- Moreover: Under multipath conditions, we are looking at multiple signals / “humps” in the spectrum; the nominal joint instantaneous bandwidth is not representative of the error made by derivative-based estimates of instantaneous frequency of the joint signal which may even exceed the range
Phase Transform

- The phase transform calculates

\[
\tilde{S}(p) = \int_0^T S(t) C(p, t) e^{-j\phi_0(p, t)} \, dt = \tilde{A}(p) e^{j\tilde{\phi}(p)}
\]

- with known “matching” amplitude C and phase \( \phi \). Bending angle profiles are obtained by

\[
\alpha(p) = \frac{1}{k} \frac{d\tilde{\phi}(p)}{dp}
\]

- This exploits the stationary phase principle; the rapidly oscillating integrand only provides contributions to the integral in the neighbourhood of stationary points (i.e., zero frequency).

- The above works for single stationary points only.
Phase Transform Integrand – Spectra

- PT integrand appears to be a mono-component signal, not very noisy;
- Allows for instantaneous phase calculation to obtain its stationary points
For impact parameters affected by multipath, the PT integrand exhibits multiple stationary points...

...so the phase transform won’t work.

But: Number of stationary points may serve as “multipath index”
A Multipath Index

- Good agreement with results from the visual inspection of spectra & fast
- Allows flagging of bending angle levels potentially affected by impact multipath
- Identifying relevant stationary points might help in determining signal cut-off
- Absolute number depend on technical details (like smoothing, unwrapping), but overall characteristics are well represented over a broad range of parameters.
Where?

- Sokolovsky et al. (2014) showed qualitatively via simulations that deep occultation signals might be caused by horizontally localised (i.e., not spherically symmetric) ducting layers.

- 600 occultations with largest overall multipath index (out of ~9000) for each satellite
- 2 weeks (15-30 Jun 2016)
Summary

- A large number of GRAS occultations (10-15% with MPI>10) affected by impact multipath
- Impact multipath can be identified in the spectra...
- ...and also via the occurrence of multiple stationary points in the phase transform integrand...
- ...and certainly invalidates present day’s wave optics methods.
  - Are horizontal gradients the atmospheric multipath of wave optics?
  - Large structural uncertainties in tropospheric RO due to impact multipath?

- We propose a “Multipath Index” based on phase transform integrand.

- We are not convinced that spectral bandwidth is a good measure of bending angle uncertainty under impact multipath conditions, unless individual components are well separated (and even then – the window part should be addressed).